Seat No. ________________

HAK-003-1163002

M. Sc. (Sem. III) (CBCS) Examination

May – 2023

Mathematics : CMT-3002

(Functional Analysis)

Faculty Code : 003
Subject Code : 1163002

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions :

(1) There are total five questions.
(2) All questions are mandatory.
(3) Each question carries equal marks.

1 Answer any **seven** of the following : **7×2=14**

(1) Define with example : Banach Space.
(2) Define with example : Quotient norm.
(3) State Holder's Inequality.
(4) Define with example : Dula Space.
(5) Define with example : Equivalent norms.
(6) True or false ? Justify $(l^{\infty}, \|\cdot\|_{\infty})$ has a Schauder basis.
(7) Define :
  (i) Cauchy sequence in normed space.
  (ii) Compactness of a subset of a metric space.
(8) Define with example : Hilbert space.
(9) State and prove parallelogram law for inner product space.
(10) Define : Weakly convergent sequence.

2 Answer any **two** of the following : **2×7=14**

(1) State and prove Minkowski's inequality.
(2) Let X be normed linear space over $\mathbb{K}$. If every absolutely convergent series in $(X, \|\cdot\|)$ converges in $(X, \|\cdot\|)$ then prove that $(X, \|\cdot\|)$ is a Banach space.
(3) Show that every finite dimensional vector subspace Y of normed linear space X over $\mathbb{K}$ is a Banach space.

3 Answer the following : **2×7=14**

(1) If Y is a closed and bounded subset of a finite dimensional normed linear space X over $\mathbb{K}$ then prove that Y is compact.

(2) State and prove Riesz lemma.

OR

3 Answer the following : **2×7=14**

(1) State and prove Uniform Boundedness Principle.

(2) State and prove Hahn Banach theorem for normed linear space over K.

4 Answer the following : **2×7=14**

(1) In an inner product space X, if $x_n \to x$, and $y_n \to y$, then prove that $\langle x_n, y_n \rangle \to \langle x, y \rangle$.

(2) State and prove Schwartz inequality for inner product space.

5 Answer any **two** of the following : **2×7=14**

(1) Define reflexive space and prove every Hilbert space is reflexive.

(2) State and prove Riesz representation theorem for bounded linear functional on Hilbert spaces.

(3) Show that

$$l^p = \left\{ (x_1, \ldots\ldots, x_n, \ldots) : x_n \in K, \forall n = 1, 2, \ldots \ \& \ \sum_{n=1}^{\infty} |x_n|^p < \infty \right\} \text{ with}$$

$\|\cdot\|_p : l^p \to \mathbb{R}$ defined by

$$\|(x_1, \ldots, x_n \ldots)\|_p = \left(\sum_{n=1}^{\infty} |x_n|^p \right)^{\frac{1}{p}}, \forall (x_1, \ldots\ldots, x_n, \ldots\ldots) \in l^p$$

is norm on l^p and $(l^p, \|\cdot\|_p)$ is a Banach space.

(4) Let $X_1, \ldots\ldots, X_n$ be a norm linear space over K. Then show that $(X_1 \times \ldots \times X_n, \|\cdot\|)$ is a Banach space over K if and only if X_1 is a Banach space over $K, \forall i = 1, \ldots, n$.

Where $\|(x_1, \ldots, x_n\| = \max_{1 \le i \le n} \|x_i\|, \forall (x_1, \ldots, x_n) \in X_1 \times \ldots \times X_n$.